Table 13-1. Maxwell's Equations, General Set

Point Form	Integral Form
$\nabla \times \mathbf{H} = \mathbf{J}_c + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J}_c + \dfrac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{S}$ (Ampère's law)
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\dfrac{\partial \mathbf{B}}{\partial t}\right) \cdot d\mathbf{S}$ (Faraday's law; S fixed)
$\nabla \cdot \mathbf{D} = \rho$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho \, dv$ (Gauss' law)
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$ (nonexistence of monopole)

For free space, where there are no charges ($\rho = 0$) and no conduction currents ($\mathbf{J}_c = 0$), Maxwell's equations take the form shown in Table 13-2.

Table 13-2. Maxwell's Equations, Free-Space Set

Point Form	Integral Form
$\nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\dfrac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{S}$
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\dfrac{\partial \mathbf{B}}{\partial t}\right) \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{D} = 0$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = 0$
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$

The first and second point-form equations in the free-space set can be used to show that time-variable $\mathbf{E}$ and $\mathbf{H}$ fields cannot exist independently. For example, if $\mathbf{E}$ is a function of time, then $\mathbf{D} = \epsilon_0 \mathbf{E}$ will also be a function of time, so that $\partial \mathbf{D}/\partial t$ will be nonzero. Consequently, $\nabla \times \mathbf{H}$ is nonzero, and so a nonzero $\mathbf{H}$ must exist. In a similar way, the second equation can be used to show that if $\mathbf{H}$ is a function of time, then there must be an $\mathbf{E}$ field present.

The point form of Maxwell's equations is used most frequently in the problems. However, the integral form is important in that it better displays the underlying physical laws.

Solved Problems

13.1. In region *1* of Fig. 13-4, $\mathbf{B}_1 = 1.2\mathbf{a}_x + 0.8\mathbf{a}_y + 0.4\mathbf{a}_z$ (T). Find $\mathbf{H}_2$ (i.e., $\mathbf{H}$ at $z = +0$) and the angles between the field vectors and a tangent to the interface

Write $\mathbf{H}_1$ directly below $\mathbf{B}_1$. Then write those components of $\mathbf{H}_2$ and $\mathbf{B}_2$ which follow directly from the two rules $\mathbf{B}$ *normal is continuous* and $\mathbf{H}$ *tangential is continuous* across a current-free interface.

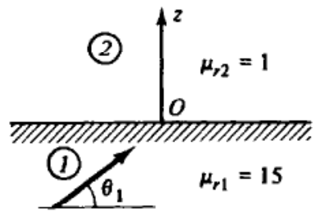

Fig. 13-4

$$\mathbf{B}_1 = 1.2\mathbf{a}_x + 0.8\mathbf{a}_y + 0.4\mathbf{a}_z \quad \text{(T)}$$

$$\mathbf{H}_1 = \frac{1}{\mu_0}(8.0\mathbf{a}_x + 5.33\mathbf{a}_y + 2.67\mathbf{a}_z)10^{-2} \quad \text{(A/m)}$$

$$\mathbf{H}_2 = \frac{1}{\mu_0}(8.0\mathbf{a}_x + 5.33\mathbf{a}_y + 10^2\mu_0 H_{z2}\mathbf{a}_z)10^{-2} \quad \text{(A/m)}$$

$$\mathbf{B}_2 = B_{x2}\mathbf{a}_x + B_{y2}\mathbf{a}_y + 0.4\mathbf{a}_z \quad \text{(T)}$$

Now the remaining terms follow directly:

$$B_{x2} = \mu_0\mu_{r2}H_{x2} = 8.0 \times 10^{-2}\,\text{(T)} \qquad B_{y2} = 5.33 \times 10^{-2}\,\text{(T)} \qquad H_{z2} = \frac{B_{z2}}{\mu_0\mu_{r2}} = \frac{0.4}{\mu_0}\,\text{(A/m)}$$

Angle θ_1 is $90° - \alpha_1$, where α_1 is the angle between $\mathbf{B}_1$ and the normal, $\mathbf{a}_z$.

$$\cos\alpha_1 = \frac{\mathbf{B}_1 \cdot \mathbf{a}_z}{|\mathbf{B}_1|} = 0.27$$

whence $\alpha_1 = 74.5°$ and $\theta_1 = 15.5°$. Similarly, $\theta_2 = 76.5°$.

Check: $(\tan\theta_1)/(\tan\theta_2) = \mu_{r2}/\mu_{r1}$.

13.2. Region *1*, for which $\mu_{r1} = 3$, is defined by $x < 0$ and region *2*, $x > 0$, has $\mu_{r2} = 5$. Given

$$\mathbf{H}_1 = 4.0\mathbf{a}_x + 3.0\mathbf{a}_y - 6.0\mathbf{a}_z \quad \text{(A/m)}$$

show that $\theta_2 = 19.7°$ and that $H_2 = 7.12$ A/m.

Proceed as in Problem 13.1.

$$\mathbf{H}_1 = 4.0\mathbf{a}_x + 3.0\mathbf{a}_y - 6.0\mathbf{a}_z \quad \text{(A/m)}$$

$$\mathbf{B}_1 = \mu_0(12.0\mathbf{a}_x + 9.0\mathbf{a}_y - 18.0\mathbf{a}_z) \quad \text{(T)}$$

$$\mathbf{B}_2 = \mu_0(12.0\mathbf{a}_x + 15.0\mathbf{a}_y - 30.0\mathbf{a}_z) \quad \text{(T)}$$

$$\mathbf{H}_2 = 2.40\mathbf{a}_x + 3.0\mathbf{a}_y - 6.0\mathbf{a}_z \quad \text{(A/m)}$$

Now

$$H_2 = \sqrt{(2.40)^2 + (3.0)^2 + (-6.0)^2} = 7.12\ \text{A/m}$$

The angle α_2 between $\mathbf{H}_2$ and the normal is given by

$$\cos\alpha_2 = \frac{H_{x2}}{H_2} = 0.34 \qquad \text{or} \qquad \alpha_2 = 70.3°$$

Then $\theta_2 = 90° - \alpha_2 = 19.7°$.

13.3. Region *1*, where $\mu_{r1} = 4$, is the side of the plane $y + z = 1$ containing the origin (see Fig. 13-5). In region *2*, $\mu_{r2} = 6$. $\mathbf{B}_1 = 2.0\mathbf{a}_x + 1.0\mathbf{a}_y$ (T), find $\mathbf{B}_2$ and $\mathbf{H}_2$.